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Real-time Animation of Sand-Water Interaction

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Abstract

Recent advances in physically-based simulations have made it possible to generate realistic animations. However, in the case of solid-fluid coupling, wetting effects have rarely been noticed despite their visual importance especially in interactions between fluids and granular materials.
This paper presents a simple particle-based method to model the physical mechanism of wetness propagating through granular materials; Fluid particles are absorbed in the spaces between the granular particles and these wetted granular particles then stick together due to liquid bridges *that are caused by surface tension and which will subsequently disappear when over-wetting occurs. Our method can handle these phenomena by introducing a* wetness value *for each granular particle and by integrating those aspects of behavior that are dependent on wetness into the simulation framework. Using this method, a GPU-based simulator can achieve highly dynamic animations that include wetting effects in real time.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

1. Introduction

Almost anyone who has spent time at the beach as a child has no doubt played with sand. When wet, sand becomes *sticky* enough to create sculptures of various shapes and sizes. These works of art then often crumble away under the onslaught of waves, never to be seen again.

Physically-based simulations are becomimg increasingly popular in computer graphics, with realistic animations of rigid bodies, fluids and soft bodies all becoming possible. Unfortunately, those fond memories of the beach cannot be completely recreated as yet. That is, there are no methods that take moisture absorption into account when simulating the dynamics of granular materials such as sand.

In this paper, we propose a method that adds the effects of moisture absorption to a simulation of fluids and granular materials. We based our method on a traditional particle-based simulation framework in order to achieve dynamic animations. We use the *Discrete Element Method* (DEM) [CS79] to simulate granular materials and *Smoothed Particle Hydrodynamics* (SPH) [Luc77,GM77] for fluids. At the interface between the fluid and the granular particles, the fluid is absorbed into the gaps between the particles, thereby increasing their wetness. These particles then attach themselves to each other due to the liquid bridges that form between them. The liquid bridges are caused by surface tension, and disappear if the material becomes too wet, releas-

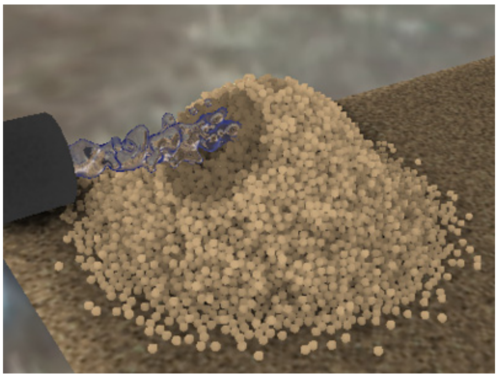


Figure 1: *A pile of sand consisting of 16k particles. The frame rate is about 30 fps.*

ing the bond between the particles. To model these physical phenomena, we introduce a *wetness value* for each granular particle, and extend the simulation framework to handle the behavior of these particles with regard to their wetness. Our proposed model is a simplified physical model, targeting games and other interactive applications. The simulator, running entirely on a GPU, can produce animated scenes such as that in Figure 1 in real time.

The rest of this paper is composed as follows: Section 2 reviews other research related to our method, Section 3 introduces the simulation algorithms that our method is based on, Section 4 describes the method in detail. After showing several experimental results in Section 6, Section 7 concludes this paper.

2. Related Work

Physically-based simulation has a long history in computer graphics. In this section, we introduce some of the many techniques that have been proposed which relate to fluid and granular simulations, along with the interactions between different types of materials.

2.1. Fluids

Fluid simulations are divided into the Eulerian and Lagrangian methods; we will focus on the latter. Lagrangian methods approximate a continuous fluid using a set of particles and model their behavior. Unlike the situation with Eulerian methods, there is no numerical dispersion, making Lagrangian techniques useful for simulations which include large topological changes in the fluid interface. In the field of computer graphics, *Smoothed Particle Hydrodynamics* (SPH) [MCG03] and the *Moving Particle Semi-implicit* (MPS) method [KO96, PTB*03] are already well-known. The former solves the particle advection equation explicitly, while the latter solves it implicitly. The former seems to be the preferred method for computer graphics due to its computational simplicity.

Following the proposal to apply SPH-based fluid simulations to interactive applications by Müller *et al.* [MCG03], various other methods, e.g. viscoelastic fluids [CBP05] and multiple interacting fluids [MSKG05] have already been presented. Techniques to accelerate SPH make use of hierarchical data structures [KW06], adaptive sampling density [APKG07] or GPU-based computation [HKK07]. Our method also uses SPH for the fluid simulation.

2.2. Granular Materials

The dynamics of materials such as sand, gravel or grain are usually represented either as a continuum or as a set of individual particles.

For the continuum approach, several methods use height fields [LM93, SOH99, ON03] or handle the material as a fluid [ZB05]. However, representing dynamic scenes including the scattering of particles is difficult when using these methods. Bell *et al.* [BYM05] used the *Discrete Element Method* (DEM) [CS79] to represent scenes as separate particles, and Harada [Har07] implemented a DEM simulation on the GPU. We use DEM in order to be able to handle dynamic animations.

2.3. Coupling Methods

Some methods handle the interactions of different material types such as fluids and rigid bodies [CMT04, BBB07, APKG07] or fluids and soft bodies [CGFO06].

Liu *et al.* [LZLW05] used height fields and the *volume of fluid* method to model the case of fluids on the surface of an object being absorbed and causing erosion. However, their method cannot represent small-scale movements or complex changes in the object's topological structure.

Wojtan *et al.* [WCMT07] handled the animation of natural phenomena such as erosion, sedimentation, and acidic corrosion. They provided an example in which a sandcastle is washed away, but while the volume of sand decreases, there are no signs of scattering of the sand or of water absorption.

By using a particle-based method, we can represent even small-scale interactions between water and sand. Furthermore, by taking into account the wetness of the granular material when calculating its behavior, we can model phenomena such as cohesion, erosion and the absorption of water.

3. Fundamental Simulation Methods

In this section, we briefly introduce those simulation methods that form the basis of our work, i.e SPH and DEM, with special regard to the force calculations that are essential to the simulation. The interactions at the interfaces of the SPH and DEM regions are explained separately in Section 4.3.

3.1. Smoothed Particle Hydrodynamics (SPH)

SPH is a particle-based simulation method, which was originally developed for use in astronomy [Luc77,GM77]. It uses a set of particles as a discrete approximation of a continuum, expressing a field quantity $A(\mathbf{x})$ by interpolating between the respective quantities around point $\mathbf{x}$; namely A_i for each particle i, as follows:

$$A(\mathbf{x}) = \sum_i m_i \frac{A_i}{\rho_i} W(\mathbf{x} - \mathbf{x}_i, h), \tag{1}$$

where m_i is the mass of particle i, ρ_i is its density and $\mathbf{x}_i$ its position. The function $W(\mathbf{x}, h)$ is a smoothing kernel with core radius h.

When applied to fluids, each of the terms in the governing Navier-Stokes equations are expressed in the above-mentioned format. The formulaization of Müller *et al.* [MCG03], which we also use in our method, keeps forces between particles symmetric by calculating the pressure and viscosity terms as follows:

$$\mathbf{F}_i^{pressure} = -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j, h), \quad (2)$$

$$\mathbf{F}_i^{viscosity} = \mu \sum_j m_j \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j} \nabla^2 W(\mathbf{x}_i - \mathbf{x}_j, h), \quad (3)$$

where μ, p_i and $\mathbf{v}_i$ represent the viscosity coefficient, the pressure and the velocity of the particles, respectively. For more details, please refer to their paper [MCG03].

3.2. Discrete Element Method (DEM)

DEM is also a particle-based simulation method, which was originally used for rock mechanics problems [CS79]. For a pair of colliding particles i and j, it calculates the normal and tangential forces acting on the particles: $\mathbf{F}_i^{normal}$ and $\mathbf{F}_i^{tangential}$. Note that the normal direction is defined by the vector from a center $\mathbf{x}_j$ to $\mathbf{x}_i$ while the tangential direction is defined as perpendicular to the normal direction. The normal force is modeled in terms of springs and dampers between the particles, while the tangential force is due to friction.

$$\mathbf{F}_i^{normal} = \mathbf{F}_i^{spring} + \mathbf{F}_i^{damper}, \quad (4)$$

$$\mathbf{F}_i^{spring} = k_s(d - \|\mathbf{x}_i - \mathbf{x}_j\|) \frac{\mathbf{v}_{ij}^{normal}}{\|\mathbf{v}_{ij}^{normal}\|}, \quad (5)$$

$$\mathbf{F}_i^{damper} = k_d \mathbf{v}_{ij}^{normal}, \quad (6)$$

$$\mathbf{F}_i^{tangential} = k_t \frac{\mathbf{v}_i^{tangential}}{\|\mathbf{v}_i^{tangential}\|}, \quad (7)$$

where k_s is the spring constant, $d = r_i + r_j$ (r_i and r_j are the radii of particles i and j), k_d is the damper coefficient, $\mathbf{v}_{ij}^{normal}$ and $\mathbf{v}_i^{tangential}$ are the particles' relative normal and tangential velocities and k_t is the coefficient of friction.

4. Interactions between Fluids and Granular Materials with Wetting Effects

This section describes a method for computing the interactions between fluids and granular materials. First, we introduce a physical mechanism for the propagation of wetness, followed by a presentation of the outline of our method, and then we finally describe it in detail.

4.1. Background and Overview

A pile consisting of a granular material includes a vast number of small spaces between the individual partices. When such a pile comes into contact with a fluid, wetness is absorbed into the spaces and propagates through the material, mainly induced by capillary forces. Similar descriptions can be found in studies into the weathering of stones [DEJ*99] and on-surface flows [LZLW05].

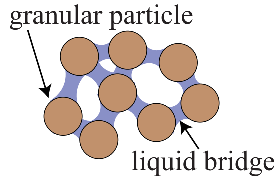


Figure 2: *Liquid bridges. For illustration purpose, the distance between particles is exaggerated.*

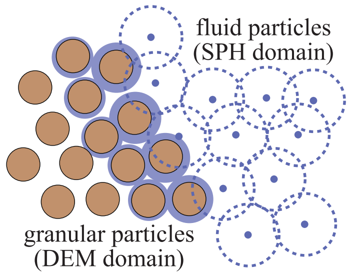


Figure 3: *Overview of our method. Wetness is provided by fluid particles, and then propagates through granular particles.*

The presence of wetness among granular particles forms structures called *liquid bridges* (Figure 2) due to the surface tension of the liquid. These liquid bridges induce grain-to-grain attractive forces and strengthen the cohesion of the material. They are therefore essential for the construction of sand castles. The force yielded by a liquid bridge can be computed in an extremely simple case (i.e. two spheres only) using a theoretical formula that agrees well with experimental data. However, in cases where there are many granular particles, the liquid-bridge forces are difficult to consider because their shapes become complicated, and, to the best of our knowledge, there are no reasonable theoretical models available as yet. Refer to the paper [Her05] for recent advances in the physics of wet granular materials.

This paper presents a simple, empirical model for the propagation of wetness and the forces yielded by wetness. In our model, each granular particle is regarded as spherical, and has an individual wetness value. We assume that the intensity of the liquid-bridge forces reduce linearly with regard to wetness. We also assume that, in the propagation of wetness, gravity has much less influence compared to the capillary forces and thus can be ignored. Our method computes the interactions of a granular particle with fluid particles and other granular particles, according to its wetness (Figure 3):

Interactions with fluid particles: Inter-particle forces are computed based on SPH. Then, if the wetness value of the granular particle **does not reach** a threshold, the granular

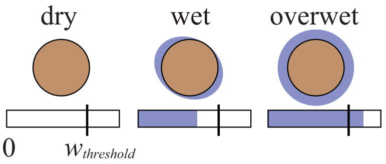
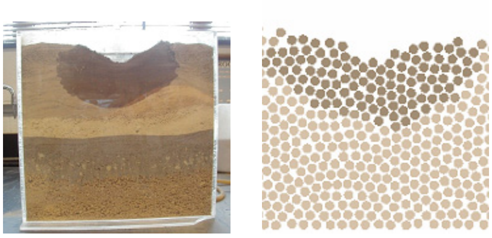


Figure 4: *Terms according to the wetness value. Each bar indicates the wetness value.*



(a) Real photo (b) 2D simulation

Figure 5: *2D comparison in which a flat surface is depressed due to moisture absorption. Photograph (http://cropsoil.psu.edu/Courses/Soils101/Labs/sec3-15.html) courtegy of Prof. Daniel D. Fritton and Katharine L. Butler.*

particle receives wetness from the fluid particles, and the fluid particles disappear (Section 4.3).

Interactions with other granular particles: Attractive forces yielded by liquid bridges are computed in addition to the forces used in DEM. Then, if the wetness value of the granular particle **exceeds** a threshold, the excessive wetness is distributed equally to those neighboring particles whose wetness values are below the threshold (Section 4.4).

Additionally, we control the propagation speed of wetness among granular particles (Section 4.5).

4.2. Wetting Model for a Granular Particle

In our model, each granular particle i has a wetness value $w_i \in [0, w_{max}]$. We refer to a granular particle by different terms according to w_i (Figure 4):

dry particle: $w_i = 0$
wet particle: $0 < w_i \leq w_{threshold}$
overwet particle: $w_{threshold} < w_i \leq w_{max}$

where $w_{threshold}$ is a threshold of wetness. Dry or wet particles receive wetness from fluid particles or overwet particles at the time of contact. The wetness value is used to compute the grain-to-grain attractive forces. In addition, in order to represent the aggregation of wet granular materials, our method shrinks the radius r_i of a granular particle i according to the wetness value:

$$r_i = R - k_r w_i, \quad (8)$$

where R is the base radius used in DEM, and k_r is a coefficient. This modification allows us to represent the depression of wet surfaces composed of granular materials. Figure 5 shows a 2D comparison between a real photo and our simulated result.

4.3. Interactions between Fluid and Granular Particles

When fluid particle i collides with granular particle j, our method first computes the inter-particle forces. The forces for fluid particle i are computed based on SPH (Section 3.1) by regarding granular particle j as a fluid particle that has a constant density and pressure, and by modifying Equations 2 and 3:

$$\mathbf{F}_{ij}^{pressure} = -\frac{V_i V_g}{4}(p_i + p_g)\nabla W(\mathbf{x}_i - \mathbf{x}_j, h), \quad (9)$$

$$\mathbf{F}_{ij}^{viscosity} = \mu \frac{V_i V_g}{2}\nabla^2 W(\mathbf{x}_i - \mathbf{x}_j, h), \quad (10)$$

where $V_i = m_i/\rho_i$ is the volume of fluid particle i, V_g is the volume of the granular particle, and p_g is a constant pressure. For granular particle j, $-\mathbf{F}_{ij}^{pressure}$ and $-\mathbf{F}_{ij}^{viscosity}$ are added as external forces.

We assume that a fluid particle has a wetness value w_{fluid}. After the computation of forces, if there are dry or wet particles in the vicinity of the fluid particle, the fluid particle is absorbed by them. That is, the fluid particle equally distributes its wetness value w_{fluid} to the dry or wet particles, then disappears.

4.4. Interactions among Granular Particles

Our method modifies the computation of forces in DEM (Section 3.2), accounting for the amount of wetness. When granular particles i, j collide with each other, the inter-particle forces are computed by modifying Equations 6 and 7:

$$\mathbf{F}_i^{damper} = k_d(1 + w_i + w_j)\mathbf{v}_{ij}^{normal}, \quad (11)$$

$$\mathbf{F}_i^{tangential} = k_t(1 + w_i + w_j)\frac{\mathbf{v}_i^{tangential}}{\left\|\mathbf{v}_i^{tangential}\right\|}. \quad (12)$$

Additionally, the liquid-bridge force $\mathbf{F}_i^{bridge}$ is computed. $\mathbf{F}_i^{bridge}$ is designed to work between wet particles only, and to reduce as the wetness increases:

$$\mathbf{F}_i^{bridge} = k_{bridge}\max\left\{0, w_f - \frac{w_i + w_j}{2}\right\}(\mathbf{v}_j - \mathbf{v}_i), \quad (13)$$

where k_{bridge} is a coefficient and w_f is a threshold for fluidization. Our method uses $\mathbf{F}_i^{bridge}$ only when the particles are moving away from each other, that is, $(\mathbf{v}_j - \mathbf{v}_i)\cdot(\mathbf{x}_j - \mathbf{x}_i) > 0$.

After computing the forces, if a granular particle i is overwet, i.e. $w_i > w_{threshold}$, and if there are dry or wet particles in its vicinity, the excessive wetness value $\Delta w = w_i - w_{threshold}$ is distributed equally among them.

4.5. Control of Propagation Speed

To control the speed of wetness propagating among granular particles, we introduce a coefficient k_p for the propagation rate. Let w_i^t be the wetness value of particle i at time t and Δw_i^t be the excessive wetness of particle i at time t (i.e., $\Delta w_i^t = w_i^t - w_{threshold}$). We assume that the propagation speed of wetness from particle i to neighboring particles j ($j = 1, 2, \ldots, N_i$) exponentially decreases according to the excessive wetness Δw_i^t:

$$w_j^{t+1} = w_j^t + k_p \frac{\Delta w_i^t}{N_i} \Delta t, \qquad (14)$$

where Δt is the timestep. Larger k_p yields faster propagation of wetness while smaller k_p results in slower propagation.

5. GPU Implementation

We implemented our model on the GPU by using a GPGPU technique [Har07] which constructs uniform grids that are used in nearest-neighbor searches. While particles in the DEM domain and the SPH domain are stored in the same grid, they can be distinguished by flags.

At the interface between fluids and granular materials (Section 4.3), fluid particles and granular particles that will interact together should be found mutually in nearest-neighbor searches. We therefore set the core radius h of the smoothing kernel used in SPH and the contact radius R used in DEM as $h = 2R$.

When equally distributing the wetness supplied by fluid particles and overwet particles, implementing such operations by random-access writes (i.e. scatter operations) is not efficient on current GPUs. Thus we perform the following two-step procedure. We store the number N_i of neighbors that particle i will supply with wetness into a texture. Then, if $N_i > 0$, the supplied wetness value is divided by N_i and added to the neighbor's wetness in a second pass.

As for rendering, granular materials are rendered as solid spheres using point sprites, and the visual quality is enhanced by a screen-space ambient occlusion technique similar to [Mit07]. Fluid is rendered as metaballs using a ray-casting technique [KSN08].

6. Results

The prototype implementation was written in C++, using OpenGL, GLSL and Cg. All experiments in this paper were conducted on a PC with an Intel Core 2 Quad 2.66GHz processor and an NVIDIA GeForce 8800 GTX graphics card. The frame rates in this paper include both simulation and rendering.

Figure 6 shows an animation sequence consisting of a pile of sand with a water stream emitted from a black faucet. This scene contains 100k fluid particles and 160k granular particles, although only a fraction of the fluid particles are actually visible. The frame rate is about 5 fps. Figure 7 is a comparison of the properties of dry, wet and overwet particles. The modified-DEM forces introduced in Section 4.4 result in different-sized piles as the particles fall to the ground. Each scene is captured at 32-60 fps, with 32k granular particles. Figure 8 shows the interaction between granular particles with a rigid shovel (with 32k particles, around 40 fps). In Figure 9, massive fluid interacts with a sand bed. This scene consists of 33k fluid particles and 64k granular particles. The frame rate is about 4 fps.

In scenes which include water, the bottleneck is the rendering of the metaballs. When we render spheres instead of metaballs for representing water, the frame rates increase to, for example, 10 fps in Figure 6 and 14 fps in Figure 9.

7. Conclusions and Future Work

We have performed simulations of interactions between fluids and granular materials based on SPH and DEM. Specifically, we have presented the following contributions in order to handle the propagation of wetness from a fluid passing through granular materials and the transition of the properties of the granular materials:

1. An empirical model for the propagation of wetness by means of introducing a wetness value for each granular particle (Section 4),
2. Shrinkage of the radii of granular particles for the aggregation (Section 4.2), and
3. Integration of the attractive force due to the amount of wetness introduced into the DEM framework (Section 4.4).

We have also demonstrated that a GPU-based simulator can achieve real-time performance (Section 6).

There are some phenomena that were not handled in our model; regeneration of fluid particles from overwet particles and drying of wet particles. Representing such effects will require considering temperature, pressure, interactions with the air or the passing of time.

We also would like to accelerate the simulation by employing an adaptive technique [APKG07] or hybrid methods with height fields [ON03]. We would also like to improve the performance and plausibility of rendering by employing e.g. subsurface scattering [JLD99] of wet materials.

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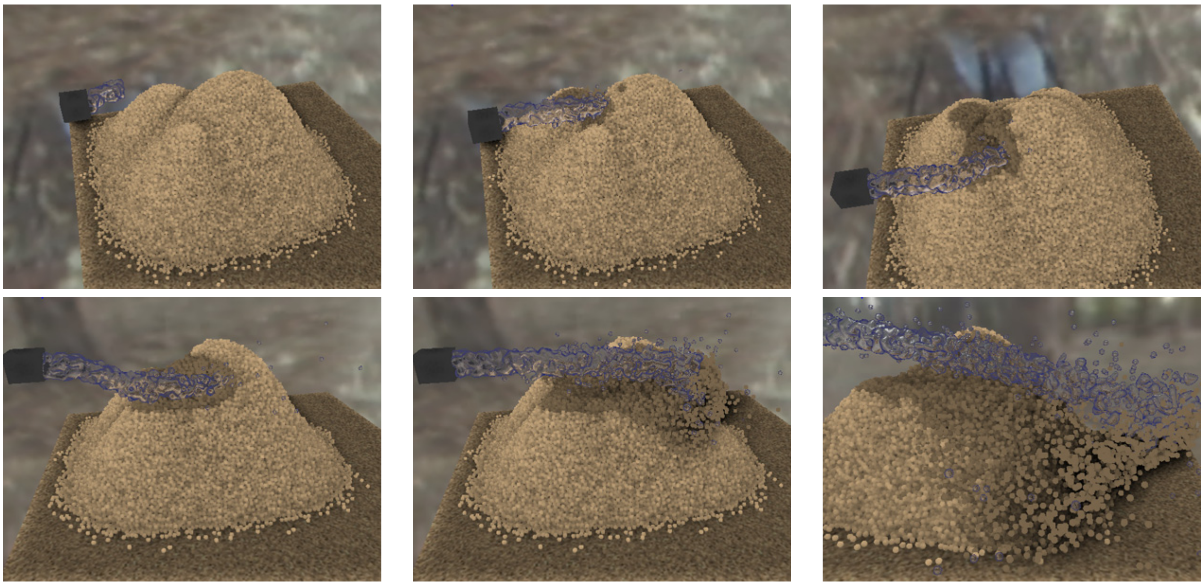

Figure 6: *Animation sequence of a pile of sand with a water stream emitted from a black faucet.*

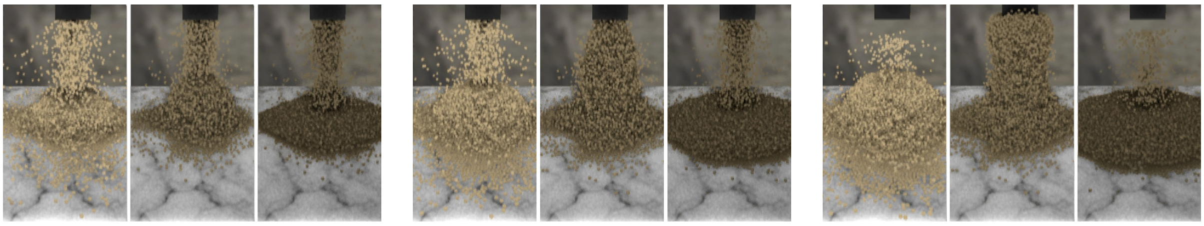

Figure 7: *Comparison with dry (light brown), wet (brown) and overwet (dark brown) particles.*

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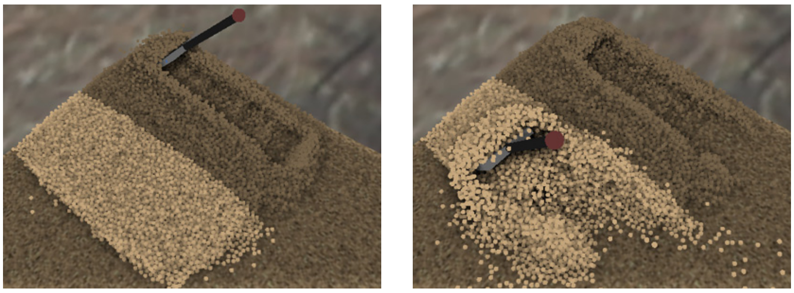

Figure 8: *Interaction between granular particles with a rigid shovel.*

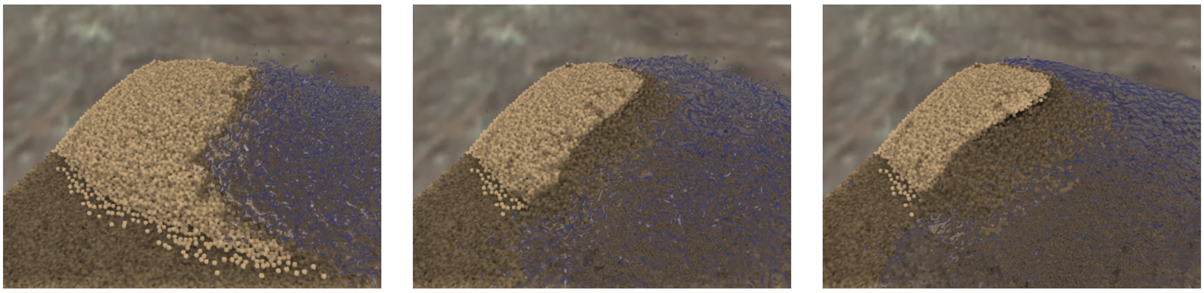

Figure 9: *Interactions between massive fluid and a sand bed.*

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